Numerical Integration

The *integral* is the area under a curve between two points on the curve. It is approximated by summing up the areas of a series of rectangles of equal width under the curve between those two points. To *integrate* is the process of finding the integral. See the figure.

For the exercises below, the x-axis between x = 0 and x = 4 is divided into to 40 equal pieces. Each piece is

$$dx = \frac{(4-0)}{40} = 0.10 \text{ units}$$

wide. The integral between the two points on a curve is approximated numerically by using the following spreadsheet:

rectangle	**x**	**y**	**dx**	**y*dx**	**sum y*dx**
1	0.00				
2	0.10				
3	0.20				
. . .					
38	3.70				
39	3.80				
40	3.90				

This spreadsheet works in general for any function y to compute the integral for the values of x ranging from 0 to 4 in steps of 0.1. The following procedure is used to create this spreadsheet:

1. Format all columns to display four decimal places.

2. Create the column for **n** for numbers 0 through 39. Going to 40 is one rectangle to many!

3. Create the column for **x** for each value of **n** .

4. Create the column for **y** for each value of **x** .

5. Create a column for **dx** for each value of **x** .

6. Create the column for **y*dx** .

7. Create the column for **sum y*dx** . An entry in this column is the cumulative sum of all the prior entries above this entry. Note the **Σ** tool in the Excel tool bar.

8. Create a chart of the graphs of **y** and **sum y*dx** plotted against **x** . Be sure to label your graphs appropriately.

1. For the function

$$y = ۲$$

use the above spreadsheet. What relationship do you see between the two graphs?

2. Repeat this exercise for

$$y = 2x$$

What relationship do you see between the two graphs?

3. Repeat this exercise for

$$y = \cos(x)$$

What relationship do you see between the two graphs?